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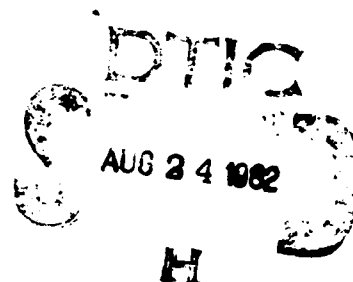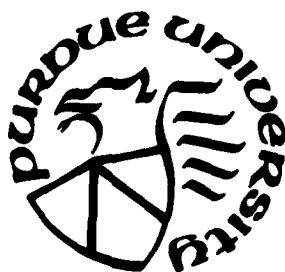
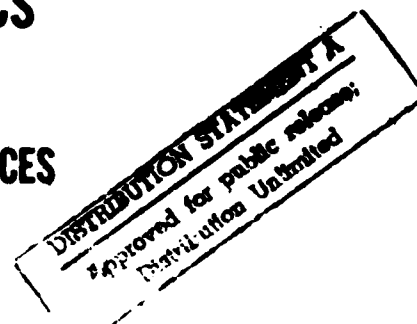## DEPARTMENT OF STATISTICS

DIVISION OF MATHEMATICAL SCIENCES

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ON ISOTONIC SELECTION RULES FOR BINOMIAL POPULATIONS
BETTER THAN A STANDARD*

by

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0. Introduction

The problem of selecting populations better than a control with
respect to a location parameter under ordering prior has been considered
in [4]. In this paper we consider the case of binomial populations
(important for discrete data) for which the parameters of interest are
not the location parameters as was the case studied in [4]. We consider
both cases when the parameter of control is known and unknown. For the
case of known control, we propose an isotonic procedure which is given
in Section 2.1. The results in this section deal with both cases,
namely, the sample sizes all equal, and unequal. Some recursive
relations are derived for computing the constants required for the
proposed procedure. When the control is unknown, a conditional
isotonic procedure is proposed in Section 2.2. This procedure provides
a conservative solution for the unconditional procedure. Brief tables of
associated constants for the proposed procedures are given in Table 1 and
Table 2.

1. Notations and Definitions

Assume $\pi_0, \pi_1, \ldots, \pi_k$ are all binomial populations such that π_i has
density $b(m; p_i)$, $i = 0, 1, 2, \ldots, k$. It is assumed that $p_1 \leq p_2 \leq \cdots \leq p_k$,
however, the actual values of these p_i's are not known. We

consider π_0 as a control and our goal is to select a subset of these
k populations so that all "good" populations are included in the subset
selected, where π_i is considered "good" if and only if $p_i \geq p_0$. By a
correct selection (CS) we mean the selection of any non-trivial subset
which contains all good populations.

Let $\Omega = \{(p_0, p_1, \ldots, p_k) | 0 < p_1 \leq p_2 \leq \ldots \leq p_k < 1, 0 < p_0 < 1\}$.
Let us denote the sets $a_i = \{i, i+1, \ldots, k\}$, $1 \leq i \leq k$ and $a_0 \equiv \phi$ (the
empty set). If action a_i is taken, it means the subset $\{\pi_i, \pi_{i+1}, \ldots, \pi_k\}$
of the k populations is selected. Since by our assumption that p_i are
ordered according to an ascending (simple) ordering prior, it is
therefore appropriate to restrict ourselves to the action space
$G = \{a_0, a_1, a_2, \ldots, a_k\}$. For given positive integer $n_0, n_1, \ldots, n_k$, we
assume n_i independent observations are drawn from π_i (i = 0,1,2,...,k)
which are, respectively, denoted by $X_{i1}, X_{i2}, \ldots, X_{in_i}$. The sample
space is denoted by

$$\chi = \{X \in I^{n_0 + n_1 + \ldots + n_k} | X = (X_{01}, X_{02}, \ldots, X_{0n_0}, X_{11}, \ldots, X_{1n_1}, \ldots, X_{k1}, \ldots, X_{kn_k}\}$$

where I denote the set of non-negative integers.

We restrict ourselves to isotonic selection procedures δ
which satisfy the P*-condition, i.e. $\inf_{\theta \in \Omega} P_\theta(CS|\delta) \geq P^*$ where P* is a
preassigned value.

A poset $(S, \leq)$ denotes a non-empty set S with a binary partial
order $\leq$ defined on it. A real-valued function f defined on a poset
$(S, \leq)$ is called isotonic if f preserves the order on S, i.e. $x \leq y$,
implies $f(x) \leq f(y)$. Let g be a real-valued function and let W be a
positive-valued function, both defined on a poset $(S, \leq)$. An isotonic

function g* on S is called an isotonic regression of g with weight W if

$\sum_{x \in S} [g(x)-g*(x)]^2 W(x)$ attains its minimum values over set of all

isotonic functions on S. It is well-known (see [2]) that there exists

one and only one isotonic regression of a given g with a given weight

W defined on S. Some algorithms such as "pool-adjacent-violators" or

the so-called "up-and-down blocks" are referred to in [1] and [2].

Let $n_0 = 0$ and $n_1 = n_2 = ... = n_k = n$ and $m = 1$. Let $\bar{X}_i$ denote

the sample mean from π_i, $i = 1, 2, ..., k$. The isotonic regression of

$\bar{X}_i$ with common weight n is a maximum likelihood estimate for

$\underline{p} = (p_1, p_2, ..., p_k)$ which is given by the following.

Theorem 1.1 ([2] p. 102): The maximum likelihood estimate for

$\underline{p} = (p_1, p_2, ..., p_k)$ with $p_1 \leq p_2 \leq ... \leq p_k$ is given by the isotonic

regression of $\bar{X}_i$ with common weight n, i.e. $\hat{\underline{X}} = (\hat{X}_{1,k}, \hat{X}_{2,k}, ..., \hat{X}_{k,k})$

minimizes $\sum_{i=1}^{k} (\bar{X}_i - p_i)^2 n$ where, by the max-min formula of [1], we

have

(1.1) $\hat{X}_{i,k} = \max_{1 \leq s \leq i} \{ \min_{s \leq t \leq k} ((\bar{X}_s + \bar{X}_{s+1} + ... + \bar{X}_t)/(t-s+1)) \}$

$= \max_{1 \leq s \leq i} \hat{\hat{X}}_{s,k}$

where

(1.2) $\hat{\hat{X}}_{s,k} = \min\{\bar{X}_s, \frac{1}{2} (\bar{X}_s + \bar{X}_{s+1}), ..., (\bar{X}_s + \bar{X}_{s+1} + ... + \bar{X}_k)/(k-s+1)\}$.

2.1 Isotonic Selection Procedure δ_1 When p_0 is Known.

a. Equal Sample Size Case

Since p_0 is known, we take $n_0 = 0$; assume $n_1 = n_2 = ... = n_k = n$.

Without loss of generality, we may consider $m = 1$, i.e. π_i is a

Bernoulli (p_i). For given positive constants $d_1, d_2, ..., d_k$

$(0 < d_i < p_0$, to be determined later), we propose the following

unconditional procedure δ_1: $\delta_1(\underline{X}) = a_{\in(\underline{X})}$, where

$\in(\underline{X}) = \min\{i \mid \hat{X}_{i,k} \geq p_0 - d_i\}$ and $\hat{X}_{i,k}$ is defined by (1.1).
Since $\delta_1(\underline{X})$ depends on vector $\underline{d} = (d_1, d_2, \ldots, d_k)$, we may denote
it by $\delta_1(\underline{d})$ when there is no confusion.

Determination of $\underline{d}$ for $\delta_1(\underline{d})$

In order to satisfy the basic P*-condition, we need to compute
$\inf_{\Omega} P(CS \mid \delta_1(\underline{d}))$. For notational conveniences, we define

$$\Omega_i = \{\underline{p} \in \Omega \mid p_{k-i} < p_0 \leq p_{k-i+1}\}, \quad i = 1, 2, \ldots, k-1$$

$$\Omega_k = \{\underline{p} \in \Omega \mid p_0 \leq p_1\}$$

and

$$\Omega_0 = \{\underline{p} \in \Omega \mid p_k < p_0\}.$$

Then, Ω_i are disjoint and $\Omega = \bigcup_{i=0}^{k} \Omega_i$. Again for notational convenience,
when there is no confusion, we denote, respectively, $\hat{X}_{i,k}$ and $\hat{\hat{X}}_{j,k}$ by
$\hat{X}_i$ and $\hat{\hat{X}}_j$ for a given fixed k. Then, for any $\underline{p} \in \Omega_i$,

$$(2.1) \quad P_{\underline{p}}(CS \mid \delta_1(\underline{d})) = P_{\underline{p}}\left\{ \bigcup_{j=1}^{k-i+1} \{\hat{X}_j \geq p_0 - d_j\} \right\}$$

$$= P_{\underline{p}}\left\{ \bigcup_{j=1}^{k-i+1} \bigcup_{r=1}^{j} (\hat{\hat{X}}_r \geq p_0 - d_j) \right\}$$

$$\geq P_{\underline{p}}\{\hat{\hat{X}}_{k-i+1} \geq p_0 - d_{k-i+1}\}$$

Since $P_{\underline{p}}(\hat{\hat{X}}_{k-i+1} \geq p_0 - d_{k-i+1})$ is increasing in p_{k-i+j} for $j = 1, 2, \ldots, i$,
keeping all other (i-1) parameters fixed, hence, the right hand side
of (2.1) attains its minimum at $p_{k-i+1} = p_{k-i+2} = \ldots = p_k = p_0$. On the
other hand, if we take a special vector

$$\overset{k-i}{\overbrace{\qquad\qquad}}$$

(2.2)　$\underline{p}_0 = (p_0, 0, 0, \ldots, 0, p_0, p_0, \ldots, p_0) \in \bar{\Omega}_i$, the closure of Ω_i,

then, we see that

$$P_{\underline{p}_0}\left(\bigcup_{j=1}^{k-i+1} \bigcup_{r=1}^{j} (\hat{\hat{X}}_r \geq p_0 - d_j)\right) = P_{\underline{p}_0}(\hat{\hat{X}}_{k-i+1} \geq p_0 - d_{k-i+1})$$

since $\overset{\times}{X}_j = 0$ a.s. when $p_j = 0$ for $j = 1, 2, \ldots, k-i$. Hence,

$\underset{\Omega_i}{\inf} P(CS|\delta_1(\underline{d}))$ attains at $\underline{p} = \underline{p}_0$ as defined by (2.2). Again, since

$$(2.3) \quad \underset{\Omega}{\inf}\, P_{\underline{p}}(CS|\delta_1(\underline{d})) = \underset{1 \leq i \leq k}{\min}\{\underset{\Omega_i}{\inf}\, P_{\underline{p}}(CS|\delta_1(\underline{d}))\}$$

because for $\underline{p} \in \Omega_0$, any action in G is, according to our definitions, a

correct selection. Therefore, if for each i ($1 \leq i \leq k$) $\underset{\Omega_i}{\inf}\, P_{\underline{p}}(CS|\delta_1(\underline{d}) \geq P^*$,

then the P^*-condition holds for $\delta_1(\underline{d})$. Now,

$$(2.4) \quad \underset{\Omega_i}{\inf}\, P\{CS|\delta_1\} = P_{\underline{p}_0}\{\hat{\hat{X}}_{k-i+1} \geq p_0 - d_{k-i+1}\}$$

$$= P_{\underline{q}_0}(\hat{\hat{X}}_{1,i} \geq p_0 - \alpha_i)$$

where

$$\underline{q}_0 = \{p_0, p_0, \ldots, p_0\} \in (0,1)^i, \text{ and } \alpha_i = d_{k-i+1}.$$

It follows from (1.2) and (2.4) that

$$(2.5) \quad \underset{\Omega_i}{\inf}\, P\{CS|\delta_1(\underline{d})\} = P_r\{Y_1 \geq c_i, \tfrac{1}{2}(Y_1 + Y_2) \geq c_i, \ldots,$$

$$\tfrac{1}{i}(Y_1 + Y_2 + \ldots + Y_i) \geq c_i\}$$

where $Y_1, Y_2, \ldots, Y_i$ are i.i.d. with Y_1 being $b(n; p_0)$ and

$c_i = n(p_0 - \alpha_i)$. Hence, it suffices to compute

$$(2.6) \quad a_i(\alpha_i) \equiv P_r\{Y_1 \geq e_1, \ Y_1 + Y_2 \geq e_2, \ldots, \sum_{j=1}^{i} Y_j \geq e_i\}$$

where

$$(2.7) \quad e_j = j \ c_i = jn(p_0 - \alpha_i), \quad j = 1, 2, \ldots, i.$$

Define

$$(2.8) \quad a_j(\alpha_i) = P_r\{Y_1 \geq e_1, \ Y_1 + Y_2 \geq e_2, \ldots, \sum_{r=1}^{j} Y_r \geq e_j\} \quad j = 1, 2, \ldots, i.$$

Letting $\langle \alpha \rangle = -[-\alpha]$, i.e. the smallest integer no less than α, we have the following useful lemma.

<u>Lemma 2.1.</u> (i) $a_1(\alpha_1) = \sum\limits_{r=r_0}^{n} \binom{n}{r} p_0^r (1-p_0)^{n-r}$ where

$$r_0 = \langle n(p_0 - \alpha_1) \rangle$$

(ii) $a_j(\alpha_i) = \sum\limits_{r_1=\langle c \rangle}^{n} g(r_1)\{ \sum\limits_{r_2=\langle 2c-r_1 \rangle}^{n} g(r_2)\{ \sum\limits_{r_3=\langle 3c-r_1-r_2 \rangle}^{n} g(r_3) \ldots$

$$\{ \sum_{r_j=\langle jc-r_1-r_2 \ldots r_{j-1} \rangle}^{n} g(r_j) \}, \qquad j = 1, 2, \ldots, i$$

where

$$c = n(p_0 - \alpha_i) \quad \text{and}$$

$$g(r) = \binom{n}{r} p_0^r (1-p_0)^{n-r}.$$

Proof: To compute (i) is straightforward. To prove (ii), define

$$A_j(\alpha, \beta) = Pr\{Y_1 \geq \alpha, \ Y_1 + Y_2 \geq \alpha + \beta, \ldots, \sum_{r=1}^{j} Y_r \geq \alpha + (j-1)\beta\}$$

where $Y_1, Y_2, \ldots, Y_j$ are iid $b(n; p_0)$. Conditioning on $Y_1 = r_1$, we obtain

$$A_j(\alpha,\beta) = \sum_{r=\langle\alpha\rangle}^{n} \binom{n}{r} p_0^r (1-p_0)^{n-r} \Pr\{Y_2 \geq \alpha+\beta-r_1, \ Y_2+Y_3 \geq \alpha+2\beta-r_1,$$

$$\cdots, \ \sum_{r=2}^{j} Y_r \geq \alpha + (j-1)\beta - r_1\}$$

$$= \sum_{r_1=\langle\alpha\rangle}^{n} \binom{n}{r} p_0^r (1-p_0)^{n-r} A_{j-1}(\alpha+\beta-r_1, \beta).$$

Taking $\alpha = \beta = n(p_0-d)$ and using (i) and by mathematical induction, we obtain (ii).

Hence, for given α_i, this lemma gives a direct method of computing $a_i(\alpha_i)$. For some special values of $n(= 5(1)10)$, $p_0(=0.1(0.1)0.5)$, $P^*(= 0.90, 0.95)$ and $i(= 1(1)4)$, the smallest values of α_i satisfying $a_i(\alpha_i) \geq P^*$ are tabulated in Table. It is to be noted that $d_{k-i+1} = \alpha_i$ $(i = 1,2,\ldots,k)$ in procedure $\delta_1(\underline{d})$.

Now define,

$$a_j'(\alpha,\beta) = P\{n(p_0-\alpha) \leq Y_1 \leq n(p_0-\beta), \ 2n(p_0-\alpha) \leq Y_1+Y_2 \leq 2n(p_0-\beta),$$

$$\cdots, jn(p_0-\alpha) \leq \sum_1^j Y_r \leq jn(p_0-\beta)\}.$$

Then, from analogous arguments as in Lemma 2.1, we have

<u>Corollary 2.1</u>:
$$a_j'(\alpha,\beta) = \sum_{r_1=\langle c_1\rangle}^{[c_2]} g(r_1)\{\sum_{r_2=\langle 2c_1-r_1\rangle}^{[2c_2-r_1]} g(r_2)\{\sum_{r_3=\langle 3c_1-r_1-r_2\rangle}^{[3c_2-r_1-r_2]} g(r_3)$$

$$\cdots \{\sum_{r_j=r_{0j}}^{r_{0j}'} g(r_j)\},$$

where

$$c_1 = n(p_0-\alpha)$$

$$c_2 = n(p_0-\beta)$$

$$r_{0j} = \langle jc_1-r_1-r_2-\ldots-r_{j-1}\rangle, \quad j = 2,3,\ldots$$

$$r_{0j}' = [jc_2-r_1-r_2-\ldots-r_{j-1}].$$

b. When the sizes n_i are not necessarily equal. We also take $n_0 = 0$ since p_0 is known, and assume $m = 1$. Then the isotonic estimates in (1.1) and (1.2) become

$$\hat{\bar{X}}_{i,k} = \max_{1 \leq s \leq i} \hat{\bar{X}}_{s,k}$$

where

(2.9) $$\hat{\bar{X}}_{s,k} = \min\{\bar{X}_s, \ (n_s\bar{X}_s + n_{s+1}\bar{X}_{s+1})/(n_s+n_{s+1}),\dots,$$

$$(n_s\bar{X}_s+n_{s+1}\bar{X}_{s+1}+\dots+n_k\bar{X}_k)/(n_s+n_{s+1}+\dots+n_k)\}.$$

For our notational simplicity, we define

$$\underline{i} = k-i+1 \quad \text{and}$$

(2.10)

$$m_{i,j} = n_i+n_{i+1} +\dots+ n_{i+j-1}.$$

Then (2.5) becomes

(2.11) $$\inf_{\Omega_i} P\{CS|\delta_1\} = P\{Z_1 \geq c_1, \ Z_1+Z_2 \geq c_2,\dots,Z_1+Z_2+\dots+Z_i \geq c_i\}$$

where

$$c_1 = m_{\underline{i},1}(p_0-\alpha_i)$$

$$c_2 = m_{\underline{i},2}(p_0-\alpha_i)$$

(2.12) $$\dots\dots\dots\dots\dots$$

$$c_i = m_{\underline{i},i}(p_0-\alpha_i)$$

and $Z_1, Z_2,\dots,Z_i$ are iid with Z_j being $b(n_{k-i+j}; p_0)$. Then, (2.11) can be computed according to the following. Define

(2.13) $$b_{j,i}(\underline{c}) = P\{Z_1 \geq c_1, \ Z_1+Z_2 \geq c_2,\dots, \sum_{r=1}^{j} Z_r \geq c_j\}, \ j = 1,2,\dots,i,$$

where $\underline{c} = (c_1,c_2,\dots,c_i)$ and Z_i's are defined by (2.12). Then, we have

Theorem 2.1. $b_{j,i}(\underline{c}) = \sum\limits_{r_1 = <c_1>}^{n_{k-i+1}} g(n_{k-i+1}, r_1)\{ \sum\limits_{r_2 = <c_2 - r_1>}^{n_{k-i+2}} g(n_{k-i+2}, r_2),$

$$\ldots, \sum\limits_{r_j = r_{j0}}^{r_{j1}} g(n_{k-i+j}, r_j)\} \qquad j = 1, 2, \ldots, i;$$

where

$$g(n; r) = \binom{n}{r} p_0^r (1-p_0)^{n-r}$$

$$r_{j0} = <c_j - r_1 - r_2 - \ldots - r_{j-1}>$$

$$r_{j1} = n_{k-i+j}$$

and $c_1, c_2, \ldots, c_j$ are defined by (2.12) and (2.10).

Proof: Conditioning on $Z_1 = r_1$ in (2.13) and following analogous arguments as those in Lemma 2.1, we obtain the result.

Corollary 2.2. If for given P^*, the constants $\underline{d} = (d_1, d_2, \ldots, d_k)$ associated with $\delta_1(\underline{d})$ are so chosen that $b_{i,i}(\underline{c}) \geq P^*$ for all $i = 1, 2, \ldots, k$, then $P_{\underline{p}}(CS|\delta_1(\underline{d})) \geq P^* \quad \forall \underline{p} \in \Omega$, where $\underline{c}$ and $b_{i,i}(\underline{c})$ are defined by (2.12) and (2.13), and $d_{k-i+1} = \alpha_i$, $i = 1, 2, \ldots, k$.

Proof: The result follows from (2.3), (2.11) and (2.13).

Computations of α_i in Table 1

For given n, i, p_0 and P^*, we start by taking $v_1 = p_0 - \frac{1}{in}$. Using Lemma 2.1, we can compute $a_i(v_1)$. If $a_i(v_1) < P^*$, we take

$\alpha_i = v_1 + \frac{1}{in} = p_0$, otherwise, we take $v_2 = v_1 - \frac{1}{in} = p_0 - \frac{2}{in}$. And

compute $a_i(v_2)$. If $a_i(v_2) < P^*$, we take $\alpha_i = v_2 + \frac{1}{in} = p_0 - \frac{1}{in}$,

otherwise take $v_3 = v_2 - \frac{1}{in}$. This process continues until for the

first time $a_i(v_{r-1}) \geq P^*$ and $a_i(v_r) < P^*$. Then, we stop and take

$\alpha_i = v_{r-1} = p_0 - \frac{r-1}{in}$. The reason that each time we decrease the amount

of $\frac{1}{in}$ is that (2.5) remains unchanged as long as the value of ic_i changes by an amount less than 1, i.e. $in(p_0-\alpha_i) < 1$.

For computations of α and β such that $a_i'(\alpha,\beta) \geq P^*$ given by Corollary 2.1 are analogous.

2.2. Selection Procedure δ_2 and Conditional Selection Procedure δ_3 When p_0 is Unknown.

For simplicity, we assume $m = 1$ and $n_0 = n_1 = \ldots = n_k = n$. When p_0 is unknown, we propose the following procedures.

$$\delta_2(\underset{\sim}{x}; \underset{\sim}{u}) = a_{\epsilon_1(\underset{\sim}{x}; \underset{\sim}{u})} \qquad \text{where}$$

$$\epsilon_1(\underset{\sim}{x}; \underset{\sim}{u}) = \min\{i \mid \hat{X}_{i,k} \geq \bar{X}_0 - u_i\}$$

where

$\hat{X}_{i,k}$ is defined by (1.1) and (1.2).

We also propose a conditional procedure δ_3 as follows.

$$\delta_3(\underset{\sim}{x}; \underset{\sim}{v} \mid \underset{\sim}{t}) = a_{\epsilon_2(\underset{\sim}{x}; \underset{\sim}{v} \mid \underset{\sim}{t})} \qquad \text{where}$$

$$\epsilon_2(\underset{\sim}{x}; \underset{\sim}{v} \mid \underset{\sim}{t}) = \min\{i \mid \hat{X}_{i,k} \geq \bar{X}_0 - v_i \text{ given that } \sum_{j=0}^{i} X_j = t_i\}$$

where

$$\underset{\sim}{t} = (t_1, t_2, \ldots, t_k), \quad t_j = \sum_0^j X_r, \quad j = 1, 2, \ldots, k.$$

We note that when $(\bar{X}_0, \bar{X}_1, \ldots, \bar{X}_k)$ is observed, $\underset{\sim}{t}$ is automatically known. However, in some situations, the experimenter may have values of $\hat{X}_{i,k} - \bar{X}_0$ without knowing $(\bar{X}_0, \bar{X}_1, \ldots, \bar{X}_k)$. For example, the given data might be $\bar{X}_1 - \bar{X}_0$, $\bar{X}_2 - \bar{X}_0, \ldots, \bar{X}_k - \bar{X}_0$.

For our convenience, we denote δ_2 and δ_3, respectively, by $\delta_2(\underset{\sim}{u})$ and $\delta_3(\underset{\sim}{v} \mid \underset{\sim}{t})$ when there is no confusion.

For fixed i, and any $\underline{p} \in \Omega_i$

$$P_{\underline{p}}(CS|\delta_2(\underline{u})) = P_{\underline{p}}(\bigcup_{j=1}^{k-i+1} \bigcup_{r=1}^{j} (\hat{\tilde{X}}_r \geq \tilde{X}_0 - u_j))$$

$$\geq P_{\underline{p}}\{\hat{\tilde{X}}_{k-i+1} \geq \tilde{X}_0 - u_{k-i+1}\}.$$

For fixed p_0, the right hand side is an increasing function of p_{k-i+1} keeping all p_{k-i+j} $(2 \leq j \leq i)$ fixed. By the same arguments as in the last section, we see that the right hand side attains its minimum at $p_{k-i+1} = p_{k-i+2} = \ldots = p_k = p_0$. By choosing $\underline{p}_0$ defined by (2.2), and applying the analogous arguments, we conclude that

$$(2.14) \quad \inf_{\Omega_i} P_{\underline{p}}(CS|\delta_2(\underline{u})) = \inf_{0 \leq p_0 \leq 1} P\{Y_1 \geq Y_0-w_1, \ Y_1+Y_2 \geq 2(Y_0-w_2),$$

$$\ldots, Y_1+Y_2+\ldots+Y_i \geq i(Y_0-w_i)\}.$$

where $Y_0, Y_1, Y_2, \ldots, Y_i$ are iid such that Y_0 is $b(n; p_0)$ and

$$(2.15) \quad w_j = j \ n \ u_i.$$

Now define

$$(2.16) \quad A(i; t_i, u_i) = \{Y_1 \geq Y_0-w_1, \ Y_1+Y_2 \geq 2(Y_0-w_2), \ldots, Y_1+Y_2+\ldots+Y_i \geq i(Y_0-w_i),$$

$$\sum_{j=0}^{i} Y_j = t_i\}$$

$$(2.17) \quad B(i; u_i) = \{Y_1 \geq Y_0-w_1, \ Y_1+Y_2 \geq 2(Y_0-w_2), \ldots, Y_1+Y_2+\ldots+Y_i \geq i(Y_0-w_i)\}$$

where w_i is defined by (2.15). Then, we have

$$(2.18) \quad P(B(i; u_i)) = \sum_{t_i=0}^{n(i+1)} P(B(i; \underline{u})|\sum_{j=0}^{i} Y_j = t_i) P(\sum_{j=0}^{i} Y_j = t_i)$$

$$= \sum_{t_i=0}^{n(i+1)} \frac{P(A(i; t_i, \underline{u}))}{\binom{n(i+1)}{t_i} p_0^{t_i} (1-p_0)^{ni+n-t_i}} P(\sum_{j=0}^{i} Y_j = t_i).$$

If for any $0 \leq p_0 \leq 1$, and any t_i $(0 \leq t_i \leq (i+1)n)$

$$(2.19) \quad P(A(i; t_i, u_i)) \geq P*\binom{n(i+1)}{t_i} p_0^{t_i} (1-p_0)^{ni+n-t_i}$$

holds, then it follows from (2.14), (2.18) and (2.19) that

(2.20) $\inf\limits_{\Omega_i} P_{\underline{p}}(CS|\delta_2(\underline{u})) \geq P*.$

Define

(2.21) $\psi_i(t,s,u) = \{(x_1,x_2,\ldots,x_i)|x_1 \geq s-nu,\ x_1+x_2 \geq 2s-2nu,$

$$\ldots,\ \sum_1^{i-1} x_j \geq (i-1)s-(i-1)nu, \sum_1^i x_j=t\} \subset I^i$$

(2.22) $q_i(t,s,u) = \sum\limits_{\psi_i(t,s,u)} \binom{n}{x_1}\binom{n}{x_2}\ldots\binom{n}{x_i}$

(2.23) $\phi_i(t,u) = \{(x_0,x_1,x_2,\ldots,x_i)|x_1 \geq x_0-nu,\ x_1+x_2 \geq 2x_0-2nu$

$$\ldots \sum_{j=1}^i x_j \geq ix_0-inu,\ \sum_{j=0}^i x_j=t\}$$

(2.24) $g_i(t,u) = \sum\limits_{\phi_i(t,u)} \binom{n}{x_0}\binom{n}{x_1}\ldots\binom{n}{x_i}.$

Then, we have

(2.25) $P(A(i;\ t_i,u))/\binom{ni+n}{t_i}p_0^{t_i}(1-p_0)^{ni+n-t_i} = g_i(t_i,u)/\binom{ni+n}{t_i}.$

It follows from (2.19) and (2.25) that in order to find u_i so that $\delta_2(\underline{u})$ satisfies the P*-condition, it suffices to find u_i such that for all t_i $(0 \leq t_i \leq n(i+1))$

(2.26) $g_i(t_i,\ u_i) \geq P*\binom{ni+n}{t_i}$

holds.

To compute $g_i(t,u)$ for given t and u, we may apply the following theorem:

Define

$$(2.27) \quad \xi_i(t,\alpha,\beta) = \{(x_1,x_2,\ldots,x_i) \mid x_1 \geq \alpha,\ x_1+x_2 \geq \alpha+\beta,\ldots,\ \sum_1^{i-1} x_j$$

$$\geq \alpha+(i-2)\beta,\ \sum_1^i x_j = t\} \subset I^i.$$

$$(2.28) \quad \zeta_i(t,\gamma) = \{(x_0,x_1,\ldots,x_i) \mid x_1 \geq x_0-\gamma,\ x_1+x_2 \geq 2(x_0-\gamma),\ldots,$$

$$\sum_1^i x_j \geq i(x_0-\gamma),\ \sum_0^i x_j = t\}$$

$$(2.29) \quad u_i(t,\alpha,\beta) = \sum_{\xi_i(t,\alpha,\beta)} \binom{n}{x_1}\binom{n}{x_2}\ldots\binom{n}{x_i}$$

$$(2.30) \quad v_i(t,\gamma) = \sum_{\zeta_i(t,\gamma)} \binom{n}{x_0}\binom{n}{x_1}\ldots\binom{n}{x_i}.$$

Then, we have the following

__Theorem 2.2:__ (i) $u_i(t,\alpha,\beta) = \displaystyle\sum_{r_1=\langle\alpha\rangle}^n \binom{n}{r_1}\{\sum_{r_2=\langle\alpha+\beta-r_1\rangle}^n \binom{n}{r_2}\{\ldots$

$$\{\sum_{r_j=\lambda_j}^n \binom{n}{r_j}\ldots\{\sum_{r_{i-1}=\lambda_{i-1}}^{\lambda} \binom{n}{r_{i-1}}\binom{n}{t-r_1-r_2-\ldots-r_{i-1}}\}\},$$

where

$$\lambda_j = \langle\alpha+(j-1)\beta-r_1-r_2-\ldots-r_{j-1}\rangle \qquad j = 2,3,\ldots,i-1,$$

$$\lambda = \min\{n,\ t-r_1-r_2-\ldots-r_{i-2}\}$$

(ii) $v_i(t,\gamma) = \displaystyle\sum_{s=s_1}^{s_2} \binom{n}{s}u_i(t-s,\alpha,s-\gamma)$ where

$$s_1 = \max\{0,\ t-in\}$$

$$s_2 = \min\{n,\ t,\ [\tfrac{t+i\gamma}{i+1}]\}$$

and $u_i(t-s,\alpha,s-\gamma)$ is given by (i).

Proof: To show (i), we note that by conditioning on $x_1 = r_1$ in the set $\xi_i(t,\alpha,\beta)$, and the lower bound of r_1 is at least α, we thus obtain $u_i(t,\alpha,\beta) = \sum\limits_{r_1=<\alpha>} \binom{n}{r_1} u_{i-1}(t-r_1, \alpha+\beta-r_1,\beta)$. Using mathematical induction and noting that $u_2(t,\alpha,\beta) = \sum\limits_{r=<\alpha>}^{r_0} \binom{n}{r}\binom{n}{t-r}$ where r_0 should exceed neither n nor t-s, we thus obtain (i).

To show (ii), we condition on the value of x_0 by taking $x_0 = s$ in the set $\zeta_i(t,\gamma)$. Then, in the set $\zeta_i(t,\gamma)$, we have simultaneously the conditions $\sum\limits_1^i x_j \geq i(s-\gamma)$ and $\sum\limits_1^i x_j = t-s$. In order that $\zeta_i(t,\gamma)$ is non-empty, we should have $t-s \geq i(s-\gamma)$ and this determines the upper bound of s. On the other hand, we also note that s should exceed neither n or t and this concludes the value s_2 in (ii). For the lower bound, we note that s should not be less than t-in if this is positive. Finally, we note that the two conditions $\Sigma x_j \geq i(s-\gamma)$ and $\sum\limits_1^i x_j = t-s$ combine into $\sum\limits_1^i x_j = t-s$ and it concludes (ii).

This completes the proof of the theorem.

Now, by taking $\alpha = s-nu$ and $\beta = s-nu$, $\xi_i(t,s-nu,s-nu) = \psi_i(t,s,u)$, also, taking $\gamma = nu$ we have $\zeta_i(t,nu) = \phi_i(t,u)$. This concludes the following

Corollary 2.3: (i) $q_i(t,s,u) = \sum\limits_{r_1=<\alpha>}^{n} \binom{n}{r_1}\{ \sum\limits_{r_2=<2\alpha-r_1>}^{n} \binom{n}{r_2}\{\ldots$

$$\{ \sum\limits_{r_j=\lambda_j}^{n} \binom{n}{r_j})\ldots\{ \sum\limits_{r_{i-1}=\lambda_{i-1}}^{\lambda} \binom{n}{r_{i-1}}\binom{n}{t-r_1-r_2-\ldots-r_{i-1}})\},$$

where

$\alpha = s-nu$

$\lambda_j = <j\alpha-r_1-r_2-\ldots-r_{j-1}>, \qquad j = 1,2,\ldots,i-1.$

$\lambda = \min\{n, t-r_1-r_2-\ldots-r_{i-2}\}.$

(ii) $\quad g_i(t,u) = \sum\limits_{s=s_1}^{s_2} \binom{n}{s} q_i(t-s,s,u)$

where s_1 and s_2 are defined in (ii) of Theorem 2.2 by taking $\gamma = nu$.

<u>Theorem 2.3.</u> (i) If, for given $P*(0 < P* < \frac{1}{k+1})$,

$g_i(t_i, u_i) \geq P*\binom{ni+n}{t_i}$ observing that $\underline{t} = (t_1, t_2, \ldots, t_k)$, then,

$\delta_3(\underline{v}|\underline{t})$ satisfies the P*-condition, where $v_{k-i+1} = u_i$, $i = 1,2,\ldots,k$.

(ii) If, for some given $P*(0 < P* < \frac{1}{k+1})$, $g_i(t_i, u_i) \geq P*\binom{ni+n}{t_i}$ for all

$t_i = 0,1,2,\ldots,(i+1)n$ and $i = 1,2,\ldots,k$, then $\delta_2(\underline{u}')$ satisfies

the P*-condition where $u'_{k-i+1} = \max\limits_{0 \leq t \leq (i+1)n} u_i(t)$.

Proof: It follows from (2.14), (2.16), (2.25) and the definition of
$\delta_3(\underline{v}|\underline{t})$ that (i) holds. For (ii), we note that $P(A(i; t_i, u_i))$
(defined by (2.16)) is increasing in u_i and (2.26) holds for all $t_i = 0,1,$
$\ldots,(i+1)n$. Finally, it follows from (2.14), (2.18) and (2.2.5).
Finally, we define

$$\xi'_i(t;\alpha_1,\alpha_2;\beta,\gamma) = \{(x_1,x_2,\ldots,x_i)|\alpha_1 \leq x_1 \leq \alpha_2, \alpha_1+\beta \leq x_1+x_2 \leq \alpha_2+\gamma,$$

$$\ldots,\alpha_1+(i-2)\beta \leq \sum_1^{i-1} x_j \leq \alpha_2+(i-2)\gamma, \sum_1^i x_j = t\}$$

$$\zeta'_i(t;\gamma,\delta) = \{(x_0,x_1,\ldots,x_i)|x_0-\gamma \leq x_1 \leq x_0+\delta, 2(x_0-\gamma) \leq x_1+x_2 \leq$$

$$2(x_0+\delta),\ldots,i(x_0-\gamma) \leq \sum_1^i x_j \leq i(x_0+\delta), \sum_0^i x_r = t\}$$

$$u'_i(t;\alpha_1,\alpha_2;\beta,\gamma) = \sum\limits_{\xi'_i(t;\alpha_1,\alpha_2;\beta,\gamma)} \binom{n}{x_1}\binom{n}{x_2}\cdots\binom{n}{x_i}$$

$$v'_i(t;\gamma,\delta) = \sum\limits_{\zeta'_i(t;\gamma,\delta)} \binom{n}{x_0}\binom{n}{x_1}\cdots\binom{n}{x_i}.$$

By the analogous arguments, we obtain the following

__Corollary 2.4:__ (i) $u_i'(t;\alpha_1,\alpha_2;\beta,\gamma) = \sum\limits_{r_1=<\alpha_1>}^{[\alpha_2]} \binom{n}{r_1}\{ \sum\limits_{r_2=s_2}^{s_2'} \binom{n}{r_2}\{\ldots$

$\{ \sum\limits_{r_j=s_j}^{s_j'} \binom{n}{r_j}\}\ldots\{ \sum\limits_{r_{i-i}=s_{i-1}}^{\lambda} \binom{n}{r_{i-1}}\binom{n}{t-r_1-r_2-\ldots-r_{i-1}}\}\}.$

where

$\qquad s_j = <\alpha_1+(j-1)\beta-r_1-r_2-\ldots-r_{j-1}> \qquad j = 2,3,\ldots,i-1.$

$\qquad s_j' = [\alpha_2+(j-1)\gamma-r_1-r_2-\ldots-r_{j-1}]$

$\qquad \lambda = \min\{n, t-r_1-r_2-\ldots-r_{i-2}\}$

(ii) $v_i'(t;\gamma,\delta) = \sum\limits_{s=s_0}^{s_1} \binom{n}{s} u_i'(t-s;s-\gamma,s+\delta;s-\gamma,s+\delta)$

where

$\qquad s_0 = \max\{[\frac{t+i\gamma}{i+1}], t-in, 0\}$

$\qquad s_2 = \min\{<\frac{t-i\delta}{i+1}>, n, t\}.$

__Computations of__ $v_i(t_i)$ __and__ u_i'

For given n, i, P* and t_i ($0 \le t_i \le n(i+1)$), we may start by taking u = 0, then apply Corollary 2.3 to compute $g_i(t_i,0)$. If

$g_i(t_i,0) \ge P*\binom{ni+n}{t_i}$, then stop and take $u_i(t_i) = 0$. Otherwise, increase

u by an amount of $\frac{1}{in}$, i.e. $u = \frac{1}{in}$. Again, using recurrence relation,

compute $g_i(t_i,u)$. If $g_i(t_i,u) \ge P*\binom{in+n}{t_i}$, then we take $u_i(t_i) = u$.

Otherwise, we continue this process until for the first time

$g_i(t_i,u) \ge P*\binom{in+n}{t_i}$. Then, we take $v_{k-i+1}(t_{k-i+1}) = u_i(t_{k-i+1})$.

For the procedure δ_1, we take $u_i' = \max\limits_{0 \le t_i \le (i+1)n} u_i(t_i)$. Some special

values of $u_i(t_i)$ associated with $n(= 5(1)10)$, $P^*(=0.90, 0.95)$ and

$i(= 1(1)5)$ are tabulated in Table 2.

3. Some Comments on the Conditional Isotonic Rule δ_2

As it can be seen, the unconditional selection procedure δ_2

defined in section 2.2 always satisfies the P^*-condition. As a matter

of fact, it follows from (2.14) that the infimum of the probability

of a correct selection attains 1 if $u_i = 1$. On the other hand, when the

condition of total sum of observations is imposed on the event $B(i; u_i)$

defined by (2.17), it becomes the event $A(i; t_i, u_i)$ defined by (2.16),

and its probability of $A(i; t_i, u_i)$ decreases. It follows from (2.18)

and (2.25) that

$$(3.1) \qquad P(B(i;u)) = \sum_{t_i=0}^{n(i+1)} \frac{g_i(t_i, u)}{\binom{ni+n}{j}} P(\sum_{j=0}^{i} Y_j = t_i)$$

where $Y_0, Y_1, \ldots, Y_i$ are iid such that Y_0 is $b(n;p)$ for some unknown

$0 < p < 1$. Let

$$(3.2) \qquad h_i(j; u) = g_i(j, u) / \binom{ni+n}{j}, \quad j = 0, 1, 2, \ldots, (i+1)n$$

where

$0 < u < 1$.

Then, (3.1) becomes

$$(3.3) \qquad P(B(i; u)) = \sum_{j=0}^{(i+1)n} h_i(j; u) \cdot b(j; (i+1)n, p)$$

where $b(j; n; p)$ is the probability of the event $\{\sum_0^i Y_r = j\}$. If

$h_i(j; u) \geq P^*$ for all $j = 0,1,2,\ldots,(i+1)n$ and for some $0 < u < 1$, then $P(B(i; u)) \geq P^*$. However, it is not true that $h_i(j; u) \geq P^*$ for all j, when the right hand side of (3.3) is not less than P^*. As some computations show, for some j (when j is large), $h_i(j; u)$ never reaches P^* (e.g. $P^* = 0.95$) no matter how large u is. This undesirable situation fortunately, never occurs in the conditional selection procedure proposed in [3].

Table 1 d_i-values (i = 1(1)5)

In this table, the smallest d_i-values (i = 1(1)5) satisfying $a_i(d_i) \geq P*$ defined by Lemma 2.1 are tabulated for p_0 = 0.1(0.1)0.5. For values of p_0 = (0.6(0.1)0.9), the problem can be treated by considering failures instead of successes. In the table, the upper entry is associated with P* = 0.90 and the lower associated with P* = 0.95. The entries under the column of (0.a - 0.b) mean that the d_i-values are the same for all these p_0-values in the range (0.a - 0.b). A " - " means the same value as the preceding value in the same column. A " * " means the same value as the preceding value in the same row.

Table 1

d_1-values

n \ p_0	0.2	0.3	0.4	0.5
5				0.500 / –
6				0.417 / 0.500
7				0.214 / 0.500
8			0.063 / 0.400	0.250 / –
9			0.056 / 0.400	0.167 / 0.278
10			0.050 / –	0.200 / –
11		0.045 / 0.300	0.045 / –	0.227 / –
12		0.042 / 0.300	0.042 / –	0.167 / 0.250
13		0.038 / 0.300	* / –	0.192 / –
14		0.036 / –	* / –	0.143 / –
15		0.033 / –	* / –	0.167 / 0.233
16		0.031 / –	* / –	0.188 / –
17	0.029 / 0.200	* / –	* / –	0.147 / 0.206
18	0.028 / 0.200	* / –	* / –	0.167 / –
19	0.026 / 0.200	* / –	* / –	0.132 / 0.184
20	0.025 / 0.200	* / –	* / –	0.150 / 0.200

d_2-values

n	0.1-0.4	0.5
5	0.100 / –	0.300 / –
6	0.083 / –	0.417 / –
7	0.071 / –	0.214 / 0.357
8	0.063 / –	0.250 / –
9	0.056 / –	0.278 / –
10	0.050 / –	0.200 / 0.300
11	0.045 / –	0.227 / –
12	0.042 / –	0.250 / –
13	0.038 / –	0.192 / 0.269
14	0.036 / –	0.214 / –
15	0.033 / –	0.167 / 0.233
16	0.031 / –	0.188 / 0.219
17	0.029 / –	0.176 / 0.206
18	0.028 / –	0.167 / 0.222
19	0.026 / –	0.184 / –
20	0.025 / –	0.150 / 0.200

d_3-values **

n	0.1-0.4	0.4	0.5
5	0.067 / –	0.067 / 0.133	0.333 / –
6	0.056 / –	0.111 / –	0.389 / –
7	0.048 / –		0.333 / 0.381
8	0.042 / –		0.250 / 0.333
9	0.037 / –		0.296 / –
10	0.033 / –		0.200 / 0.300
11	0.030 / –		0.242 / –
12	0.028 / –		0.222 / 0.250
13	0.026 / –		0.205 / 0.282
14	0.024 / –		0.214 / –
15	0.022 / –		0.200 / 0.244
16	0.021 / –		0.188 / 0.229
17	0.020 / –		0.176 / 0.216
18	0.019 / –		0.185 / 0.222
19	0.018 / –		0.193 / –
20	0.017 / –		0.150 / 0.200

d_4-values

n	0.1-0.3	0.4	0.5
5	0.050 / –	0.100 / 0.150	0.300 / 0.400
6	0.042 / –	0.125 / –	0.375 / –
7	0.036 / –	0.107 / –	0.321 / 0.393
8	0.031 / –	0.094 / –	0.281 / 0.313
9	0.028 / –	0.083 / –	0.278 / 0.333
10	0.025 / –	0.075 / –	0.300 / 0.325
11	0.023 / –	0.682 / –	0.250 / 0.318
12	0.021 / –	0.833 / –	0.250 / 0.292
13	0.019 / –	0.577 / –	0.250 / 0.269
14	0.018 / –	0.536 / –	0.267 / 0.286
15	0.017 / –	0.050 / –	0.233 / 0.250
16	0.016 / –	0.047 / –	0.234 / 0.250
17	0.015 / –	0.059 / –	0.235 / 0.250
18	0.014 / –	0.042 / –	0.222 / 0.250
19	0.013 / –	0.053 / –	0.237 / 0.250
20	0.013 / –	0.038 / –	0.237 / 0.250

*A missing entry in this table means that the value of d_1 is the same as the associated p_0. And also the values of d_1 in the column of p_0 = 0.1 are all 0.1.

**A missing entry in this table in column of p_0 = 0.4 means the value of d_3 is the same as the associated value in column 0.1-0.4.

Table 2 $u_i(t_i)$-values ($i = 1(1)5$)

Associated with $n = (5(1)10)$, $i = (1(1)5)$ and $P* = (0.90,0.95)$

the entries are $u_i(t_i)$-values satisfying $g_i(t_i,u_i) \geq P*(\binom{(i+1)n}{t_i})$, defined

by Corollary 2.3, with $t_i = \sum_0^i x_j$, the total sum of $(i+1)$ populations.

The upper entry is for $P* = 0.90$ and the lower for $P* = 0.95$. A " * "

means the same as the preceding value in the same row and a " - "

means the same value as the preceding value in the same column. The

constant $v_i \equiv v_i(t_i)$ needed for the rule δ_3 is given by $v_i(t_i) = u_i(t_i)$.

It should be pointed out that missing entries in the table were either

not computed or have been dropped for sake of convenience.

Table 2

N = 5

i/t	1	2	3	4	5	6	7	8	9	10
1	0.400	0.600	0.400	0.600	0.800					
	-	-	-	-	-					
2	0.300	*	0.500	*	*	*	*			
	-	0.500	*	0.600	0.500	*	0.700			
3	0.267	*	*	*	*	*	0.400	0.467	0.667	
	-	0.467	-	-	-	-	-	-	-	
4	0.250	*	0.150	*	*	0.250	*	*	0.450	0.850
	-	-	-	-	-	-	-	-	-	-
5	0.240	*	*	*	*	*	*	*	*	*
	-	-	-	-	-	-	-	-	-	-

N = 6

i/t	1	2	3	4	5	6	7	8	9	10
1	0.333	0.500	0.333	0.500	0.667	0.500				
	-	-	0.667	-	-	-				
2	0.250	*	0.417	*	*	*	0.500	0.583	*	
	-	0.417	-	0.500	0.583	*	0.583	*	0.833	
3	0.222	*	0.333	0.278	0.222	*	0.333	0.278	0.389	*
	-	0.389	*	-	-	-	-	0.389	-	-
4	0.208	*	0.125	*	*	*	0.208	*	*	*
	-	-	*	-	-	-	-	-	-	0.375
5	0.200	*	*	*	*	*	*	*	*	*
	-	-	0.367	-	-	-	-	-	-	-

Table 2 (cont.)

N = 7

i \ t	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.286 / -	0.429 / -	0.286 / 0.571	0.429 / -	0.571 / -	0.429 / -	0.571 / -							
2	0.214 / -	* / 0.357	0.357 / -	* / 0.429	* / 0.500	* / *	0.429 / 0.500	0.357 / -	0.357 / 0.500	0.500 / -				
3	0.190 / -	* / 0.333	0.286 / *	0.238 / *	0.190 / -	* / -	* / 0.286	0.238 / -	0.190 / 0.333	0.333 / -	* / -	* / 0.429	0.476 / 0.619	
4	0.179 / -	* / -	0.107 / -	* / -	0.071 / *	0.036 / 0.071	* / 0.107	0.107 / -	* / 0.179	0.179 / -	* / -	0.321 / 0.464	0.464 / -	0.750 / -

N = 8

i \ t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0.125 / -	0.250 / -	0.125 / 0.375	0.250 / -	0.375 / -	0.250 / 0.500	0.375 / -	0.250 / 0.500										
2	0.125 / -	0.250 / -	* / -	* / 0.313	* / 0.375	* / *	0.313 / *	0.375 / 0.438	* / 0.375	* / *	* / -	0.500 / 0.625						
3	0.167 / -	* / 0.292	0.250 / *	0.292 / -	* / -	0.167 / *	0.250 / -	0.208 / 0.292	0.167 / *	0.292 / *	* / -	* / -	* / 0.333	0.417 / -	0.542 / -			
4	0.125 / -	0.094 / *	0.063 / 0.094	0.063 / -	0.031 / *	0.031 / -	* / -	0.063 / -	* / -	* / -	0.125 / -	* / -	* / -	0.250 / -	0.375 / -	0.500 / 0.625		
5	0.150 / -	* / -	* / 0.275	0.225 / *	0.200 / *	0.175 / 0.200	* / *	0.150 / *	* / 0.175	* / -	0.150 / -	* / 0.175	* / -	* / -	* / -	* / 0.200	0.200 / 0.275	0.275 / -

Table 2 (cont.)

N = 9

t\i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.222	0.333	0.444	0.333	0.444	0.333 / 0.555	0.444 / -	0.333 / -	0.444 / -								
2	0.167 / -	0.278 / -	* / -	* / 0.389	* / -	* / -	0.333 / *	0.389 / 0.444	* / -	0.333 / *	0.278 / -	0.389 / -	* / 0.500				
3	0.148 / -	* / 0.259	0.222 / *	* / -	* / 0.296	* / -	0.222 / *	0.185 / *	0.259 / -	* / -	0.222 / 0.259	* / -	* / -	* / -	* / -	0.370 / -	0.481 / 0.592

N = 10

t\i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0.200 / -	0.300 / -	0.400 / -	0.300 / -	0.400 / -	0.300 / -	0.400 / -	0.300 / 0.500	0.400 / -	0.300 / 0.500										
2	0.150 / -	0.250 / -	* / -	* / 0.350	* / -	0.350 / *	0.300 / *	0.350 / 0.400	0.350 / -	* / 0.450	* / 0.400	* / 0.350	* / *	* / 0.450	0.450 / 0.650					
3	0.133 / -	* / 0.233	0.200 / *	0.233 / -	0.267 / *	* / -	* / -	* / -	* / -	* / -	0.233 / -	* / -	* / -	* / -	* / -	* / 0.300	0.267 / 0.333	0.333 / -	0.433 / 0.533	
4	0.120 / -	* / -	* / 0.220	0.180 / 0.220	0.220 / -	* / -	* / -	* / -	* / -	0.025 / -	0.180 / 0.220	0.160 / 0.220	* / *	0.140 / *	0.120 / 0.160	* / 0.180	* / 0.160	0.140 / -	0.120 / -	* / 0.160

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The problem of selecting a subset containing all populations better than a standard under a simple prior ordering is considered. Both cases of the standard parameter known and unknown have been considered. When the standard parameter is unknown, the proposed procedure is conservative. Some tables of constants associated with the proposed isotonic procedures are given.

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